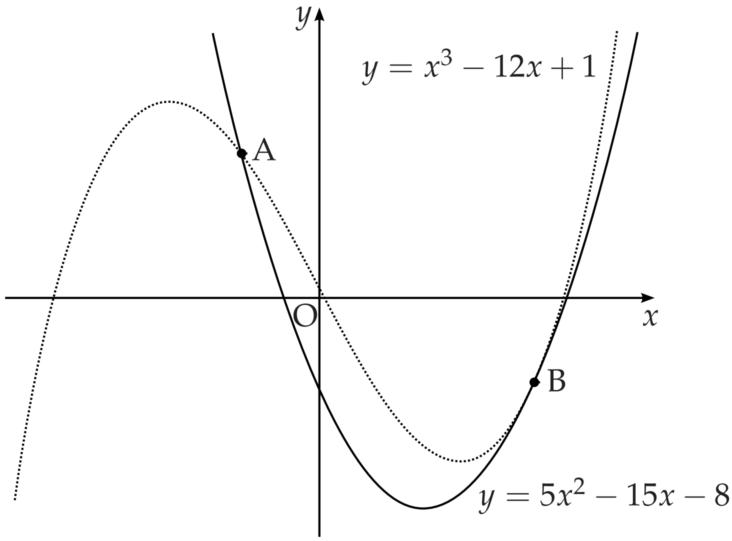
Area between two curves

[SQA] 1. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

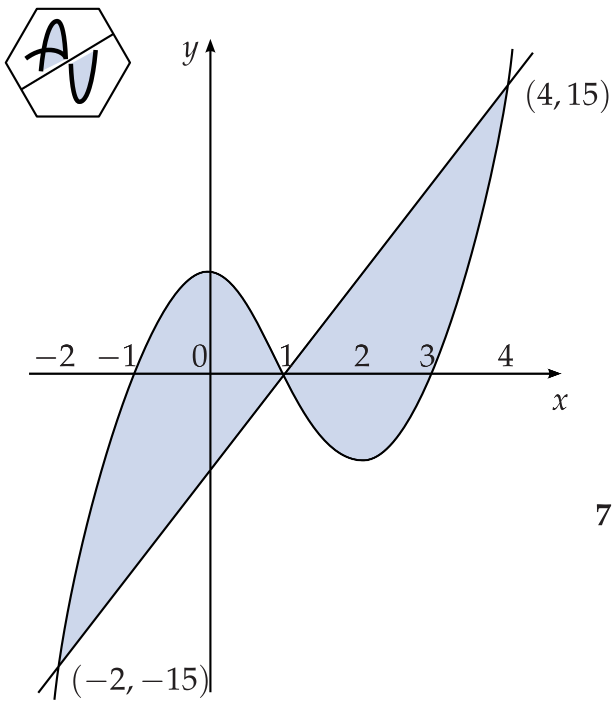
Find the area enclosed between the two curves. **5**

[SQA] 2. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area.



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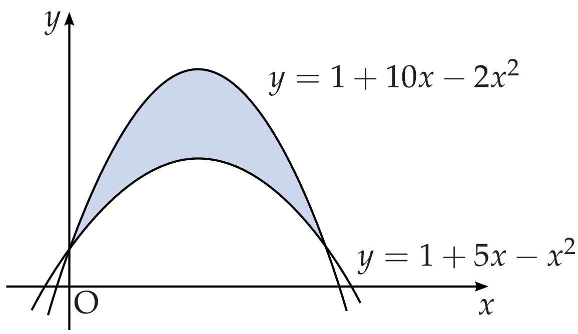
[SQA] 3. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.



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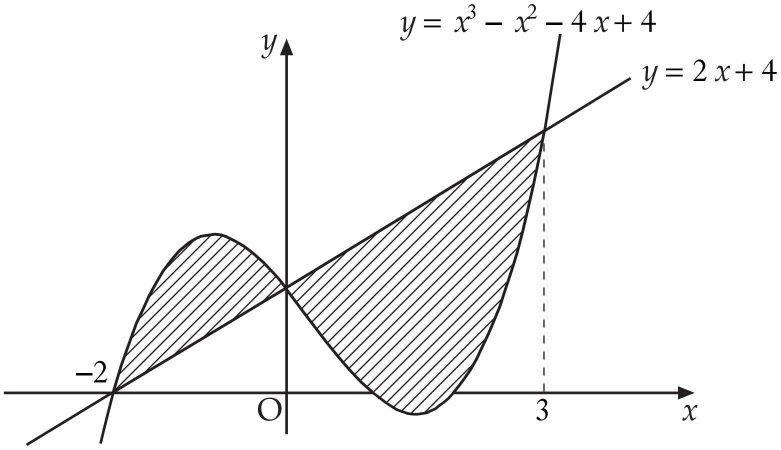
4. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.



Calculate the total shaded area. **10**

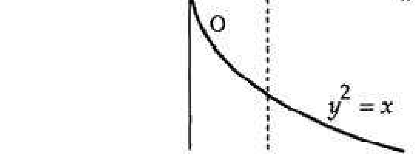
[SQA] 5.

(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. **2**

(*b*) Find the area completely enclosed between these two curves. **3**

[SQA] 6. The diagram shows two curves with equations $y = x^2$ and $y^2 = x$.

The area completely enclosed between the two curves is divided in half by the line with equation $x = k$.



(*a*) Represent these two equal areas by two separate integrals each involving k. **(6)**

(*b*) Equate the integrals and show that k is given by the equation

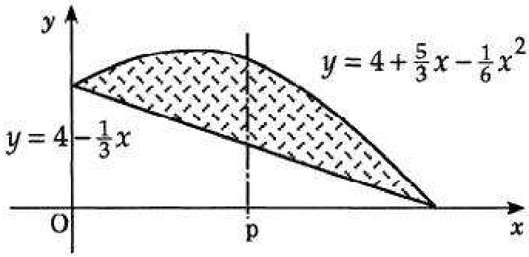
$$2k^3 - 4k^{\frac{3}{2}} + 1 = 0.$$ **(4)**

(*c*) Use the substitution p^2 for k^3 to find the value of k. **(4)**

[SQA] 7. When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.



(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**

(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.

By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

(*c*) (i) Verify that $p = 6$ is a solution of this equation.

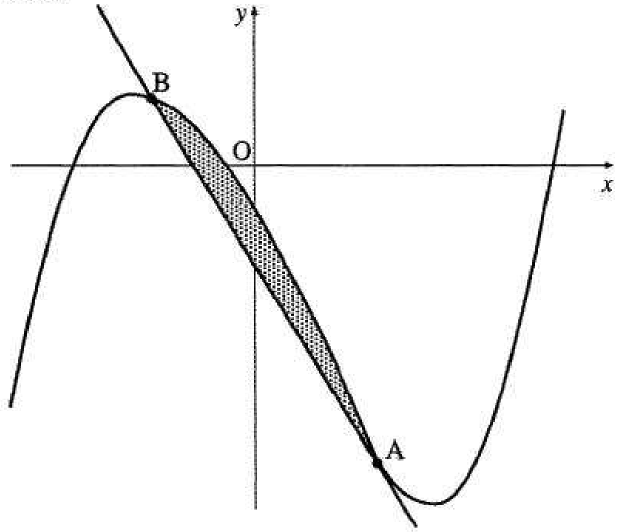
(ii) Find algebraically the other two solutions of this equation.

(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

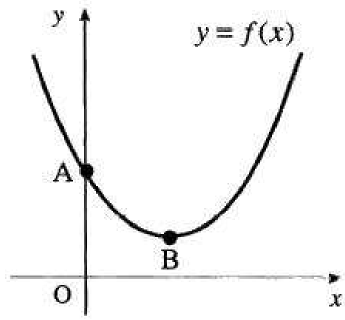
[SQA] 8. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(a) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(b) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**

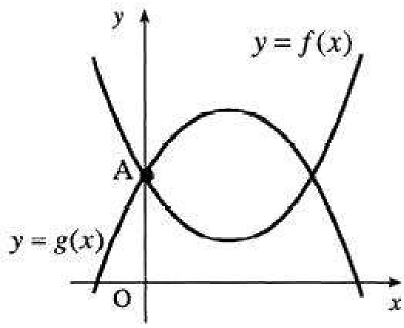


[SQA] 9. The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$. The graph cuts the y-axis at A and has a minimum turning point at B.



(a) Write down the coordinates of A and B. **(3)**

(b) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves. **(5)**



(c) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants. Write down the values of m and n. **(2)**

[SQA] 10. A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

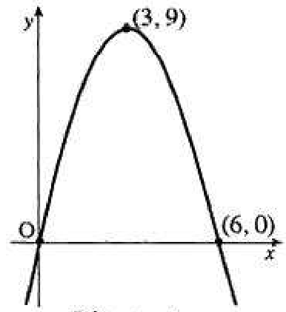


Diagram 1

(*a*) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(*b*) Find the area enclosed by the parabola and the x-axis. **(4)**

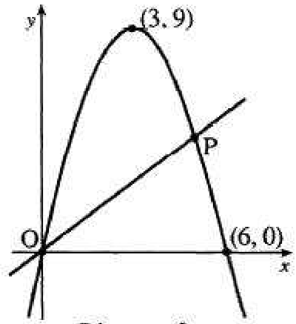


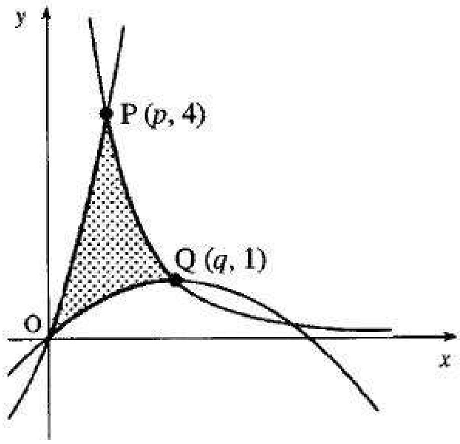
Diagram 2

(*c*) (i) Diagram 2 shows the parabola from (*a*) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

[SQA] 11. The origin, O, and the points P and Q are the vertices of a curved 'triangle' which is shaded in the diagram. The sides lie on curves with equations

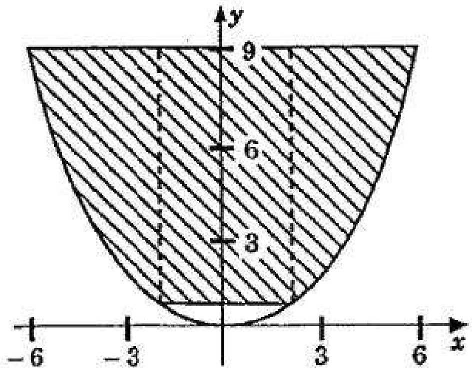
$y = x(x+3)$, $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$.



(*a*) P and Q have coordinates $(p, 4)$ and $(q, 1)$. Find the values of p and q. 2

(*b*) Calculate the shaded area. 7

[SQA] 12. **The cargo space of a small bulk carrier is 60m long.**



The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2$, $-6 \leq x \leq 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry. **(9)**

13. (*a*) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

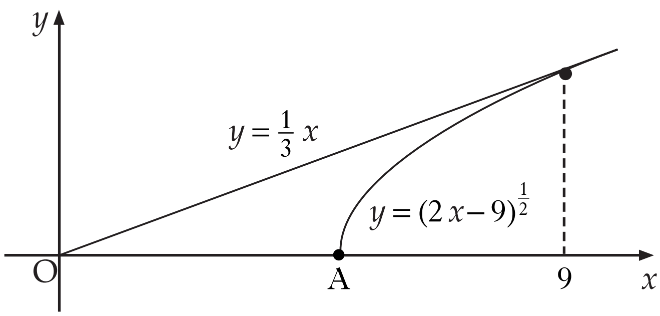


Diagram 1

Find the coordinates of point A. **1**

(*c*) Calculate the shaded area shown in diagram 2. **7**

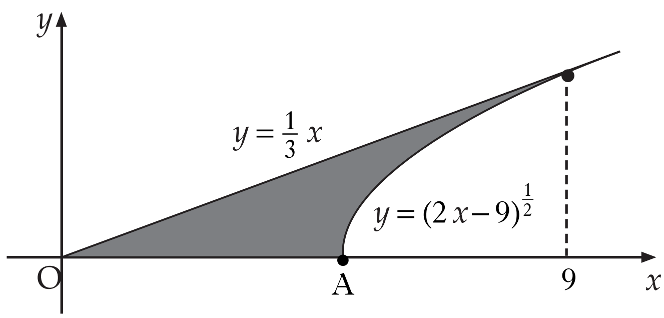
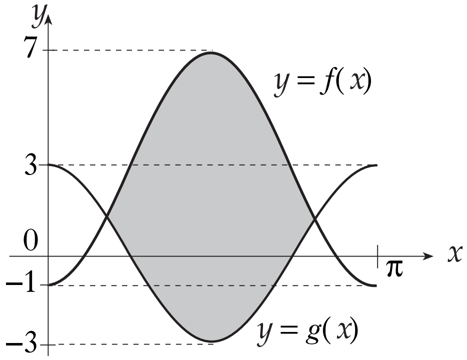


Diagram 2

[SQA] 14. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

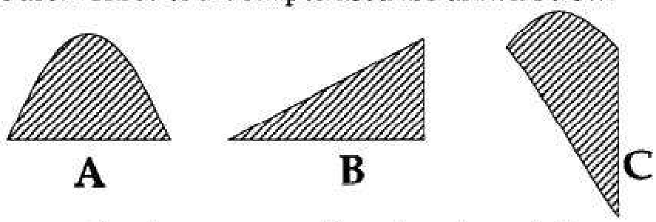


(*a*) Write down the values of m and n. **1**

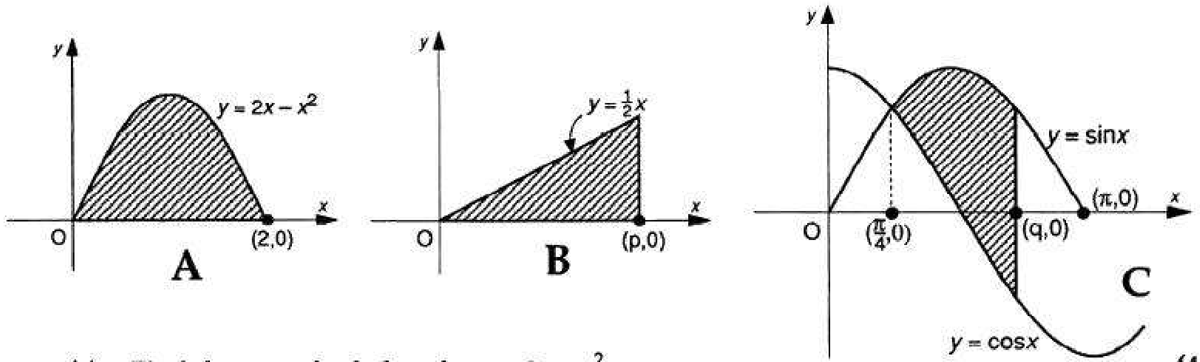
(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$. **5**

(*c*) Calculate the shaded area. **6**

[SQA] 15. An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.



Relative to x,y–axes, the shapes are positioned as shown below.



(*a*) Find the area shaded under $y = 2x - x^2$. **(4)**

(*b*) Use the area found in part (*a*) to find the value of p. **(2)**

(c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures. **(10)**

[END OF QUESTIONS]